Keynote Lectures on Microeconomics

Federico Echenique (CalTech): *Fairness and efficiency for probabilistic allocations with endowments*

John Geanakoplos (Yale): *Leverage Cycles, Credit Surfaces, and Central Banking*

Chaired by Giacomo Bonanno (UC Davis)
Fairness and efficiency for probabilistic allocations with endowments

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BGSE

NASMES — June 22, 2018
Antonio and Jun:
Discrete allocation

Agents: \( I \)
Resources: \( S \) (finite, indivisible).

\begin{align*}
1 & \quad \bullet \\
2 & \quad \bullet \\
3 & \quad \bullet \\
4 & \quad \bullet \\
5 & \quad \bullet \\
1 & \quad \bullet \\
2 & \quad \bullet \\
3 & \quad \bullet \\
4 & \quad \bullet \\
5 & \quad \bullet
\end{align*}
Agents: $I$
Resources: $S$ (finite, indivisible).
Discrete allocation

Three objectives:
- Efficiency
- Fairness
- Respect property rights

Main application: School Choice
School choice

- Property rights: school priorities
- Priorities: fairness among unequal agents
- Efficiency and fairness are incompatible.

Echenique-Miralles-Zhang

Fairness & Efficiency
School choice

- Property rights: school priorities
- Priorities: fairness among unequal agents
- Efficiency and fairness are incompatible.

This paper:

- Property rights: endowments
- Endowments: fairness among unequal agents
- Efficiency and fairness are compatible.
Priorities are difficult to map into property rights.

Endowments are explicit property rights.

For example, guarantee a:

1. chance at a good school;
2. neighborhood school;
3. slot for a sibling.
“...school choice has not delivered on a central promise: to give every student a real chance to attend a good school. Fourteen years into the system, black and Hispanic students are just as isolated in segregated high schools as they are in elementary schools — a situation that school choice was supposed to ease.”

Eric Nadelstern at Columbia University, deputy school chancellor when school choice system was implemented, proposed to have a lottery decide the allocations.

Another application:
Time banks.
Related Literature

- Justified envy w/endowments: Yilmaz (2010)
- Exog. and endog. budgets: Mas-Colell (1992) and Le (2017)
- Endowments in school choice: Hamada, Hsu, Kurata, Suzuki, Ueda, and Yokoo (2017)

More references in the paper...
Discrete allocation

Agents: \( I \)
Resources: \( S \).
Agents: $I$
Resources: $S$.

1 and 2 want school $s_1$.
Fairness: toss a coin.
Agents: \( I = \{1, \ldots, N\} \).

Schools: \( S = \{s_1, \ldots, s_L\} \).

A *lottery* is an element of

\[
\Delta_- = \{ x \in \mathbb{R}^L_+ : \sum_{j=1}^{L} x_j \leq 1 \}
\]

\( u^i : \Delta_- \rightarrow \mathbb{R} \) (cont. & mon.)
An *allocation* is $x = (x^i)_{i=1}^N$, with $x^i \in \Delta^L$, s.t.

$$\sum_{i \in I} x^i_s = q_s$$
An allocation is \( x = (x^i)_{i=1}^N \), with \( x^i \in \Delta_L \), s.t

\[
\sum_{i \in I} x^i_s = 1
\]
Envy

\[ i \text{ envies } j \text{ at } x \text{ if } u_i(x^j) > u_i(x^i) \]
\[ x^i = \left( \frac{1}{N}, \ldots, \frac{1}{N} \right) \implies \text{no envy} \]
1 and 2 would like to *trade* probability shares.
An allocation $x$ is \textit{Pareto optimal} (PO) if there is no allocation $y$ s.t

$$u^i(y^i) \geq u^i(x^i) \text{ for all } i \text{ and } u^j(y^j) > u^j(x^j)$$

for some $j$. 

Theorem (Hylland and Zeckhauser (1979))

There is a PO and envy-free allocation. It is a market equilibrium allocation.
An *HZ-equilibrium* is a pair \((x, p)\), with \(x \in \Delta^N\) and \(p = (p_s)_{s \in S} \geq 0\) s.t.

1. \(\sum_{i=1}^{N} x^i = (1, \ldots, 1)\)
2. \(x^i\) solves

\[
\text{Max} \ \{u^i(z^i) : z^i \in \Delta_- \text{ and } p \cdot z^i \leq 1\}
\]

**Condition (1): supply = demand.**
**Condition (2):** \(x^i\) is \(i\)'s demand at prices \(p\) and income = 1.

Observe:
- Income is independent of prices
- No endowments.
Suppose that each $u^i$ is linear (expected utility).

**Theorem (Hylland and Zeckhauser (1979))**

*There is a HZ equilibrium allocation. It is envy-free and PO.*
No envy makes sense for "equals."

What if agents are unequal?

In an economy with endowments, agents start from different positions.
Fairness among unequals

- Each $i$ has an *endowment* $\omega^i \in \Delta$.
- $\omega^i$ is an initial lottery.
- Suppose that $\sum_i \omega^i = (1, \ldots, 1)$.

For example, suppose schools are allocated via a lottery. Admission probabilities reflect: neighborhood school (walk-zone priority), sibling priority, or test scores.
A Walrasian equilibrium is a pair \((x, p)\) with \(x \in \Delta^N\), \(p \geq 0\) s.t

1. \(\sum_{i=1}^N x^i = \sum_{i=1}^N \omega^i\); and
2. \(x^i\) solves

\[
\text{Max} \ \{ u^i(z^i) : z^i \in \Delta_- \text{ and } p \cdot z^i \leq p \cdot \omega^i \}
\]
Proposition (Hylland and Zeckhauser (1979))

There are economies in which all agents’ utility functions are expected utility, that possesses no Walrasian equilibria.
Budget set

\[ \omega^i \]

\[ p \]

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Fairness & Efficiency
Budget set

\( \omega^i \)

(1, 1)

simplex

\( p \)
no Walras’ Law
non-responsive demand
3 agents; exp. utility

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<thead>
<tr>
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<th>$u^1$</th>
<th>$u^2$</th>
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<tbody>
<tr>
<td>$s_A$</td>
<td>10</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>$s_B$</td>
<td>1</td>
<td>1</td>
<td>10</td>
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Endowments: $\omega_i = (1/3, 2/3)$. 
3 agents; exp. utility

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Endowments: $\omega^i = (1/3, 2/3)$.

Obvious allocation:

$$x^1 = x^2 = (1/2, 1/2)$$
$$x^3 = (0, 1)$$
HZ Example

simplex
HZ Example

Obvious allocation

\[ \frac{1}{2} \quad \frac{1}{4} \quad \frac{3}{4} \]

\[ \omega^i \]
HZ Example
Moreover, . . .

- the first welfare theorem fails.
- There are Pareto ranked Walrasian equilibria.
Our results
A function \( u : \Delta^n \rightarrow \mathbb{R} \) is

- **concave** if \( \forall x, z \in \Delta_, \text{ and } \forall \lambda \in (0, 1), \)
  \[
  \lambda u(z) + (1 - \lambda)u(x) \leq u(\lambda z + (1 - \lambda)x);
  \]

- **quasi-concave** if, \( \forall x \in \Delta_-, \)
  \[
  \{ z \in \Delta_- : u(z) \geq u(x) \}
  \]
  is a convex set.

- **semi-strictly quasi-concave** if \( \forall x, z \in \Delta_-, \)
  \[
  u(z) < u(x) \text{ and } \lambda \in (0, 1) \implies u(z) < u(\lambda z + (1 - \lambda)x)
  \]

- **expected utility** if it is linear.
Let $x$ be an allocation.

- $x$ is \textit{weak Pareto optimal} (wPO) if \( \exists \) an allocation $y$ s.t \[ u^i(y^i) > u^i(x^i) \] for all $i$.

- \( \varepsilon \)-\textit{weak Pareto optimal} (\( \varepsilon \)-PO), for $\varepsilon > 0$, if \( \exists \) an allocation $y$ s.t \[ u^i(y^i) > u^i(x^i) + \varepsilon \] for all $i$. 

Let $x$ be an allocation.

- $x$ is *acceptable* to $i$ if $u^i(x^i) \geq u^i(\omega^i)$.

- $x$ is *individually rational* (IR) if it is acceptable to all agents.
i envies j at x if \( u^i(x^j) > u^i(x^i) \).

Such envy will be tolerated (i.e., not be justified) only if j’s endowment is “good enough.”
Justified envy

$i$ envies $j$ at $x$ if $u^i(x^j) > u^i(x^i)$.

Such envy will be tolerated (i.e. not be justified) only if $j$ regards $x^i$ as unacceptable.
Justified envy

\[ i \text{ envies } j \text{ at } x \text{ if } u^i(x^j) > u^i(x^i). \]

Such envy will be tolerated (i.e. not be justified) only if
\[ u^j(\omega^j) > u^i(x^i) \]
i has *justified envy* towards j at allocation x if

\[ u^i(x^j) > u^i(x^i) \text{ and } u^j(x^i) \geq u^j(\omega^j). \]
Let $x$ be an allocation.

$x$ has *no justified envy* (NJE) if no agent has justified envy towards any other agent at $x$. 
Observe: NJE and IR imply *equal treatment of equals*. 
Let $x$ be an allocation.

$x$ has *no justified envy* (NJE) if no agent has justified envy towards any other agent at $x$. 
Justified envy

- $i$ has **strong justified envy** (SJE) towards $j$ at $x$ if $u^i(x^j) > u^i(x^i)$ and $u^j(x^i) > u^j(\omega^j)$.

- For $\epsilon > 0$, $i$ has **$\epsilon$-justified envy** ($\epsilon$-JE) towards $j$ at $x$ if $u^i(x^j) > u^i(x^i)$ and $u^j(x^i) > u^j(\omega^j) - \epsilon$. 
Justified envy

no $\varepsilon$-justified envy $\implies$ no justified envy $\implies$ no strong just. envy
### Theorem

Suppose all \( u^i \) are concave, and let \( \varepsilon > 0 \).

1. \( \exists \) an allocation that is \( \varepsilon \)-IR, \( \varepsilon \)-PO and has no \( \varepsilon \)-justified envy;
2. \( \exists \) an allocation that is IR, wPO and has no strong justified envy.
Consider problem

$$\text{Max} \sum_{i} \lambda_i u^i(x_i)$$

s.t. $x$ is an allocation.

Obtain a NJE allocation from this problem

by choosing right welfare weights, $(\lambda_i) \in \Delta^N$.

(Actual proof uses an approximation to this problem, hence the $\varepsilon$).
KKM Lemma

(1, 0, 0)  (0, 1, 0)  (0, 0, 1)
KKM Lemma

\[(1, 0, 0), (0, 1, 0), (0, 0, 1)\]
KKM Lemma

\((1, 0, 0)\)

\((0, 1, 0)\)

\((0, 0, 1)\)
Proposal No. 2: Walrasian equilibrium.
Let $\alpha \in [0, 1]$

An $\alpha$-slack Walrasian equilibrium is $(x, p)$ with $x \in \Delta^N$, $p \geq 0$ s.t

1. $\sum_{i=1}^{N} x^i = \sum_{i=1}^{N} \omega^i$; and
2. $x^i$ solves

$$\text{Max} \ \{ u^i(z^i) : z^i \in \Delta_- \text{ and } p \cdot z^i \leq \alpha + (1 - \alpha)p \cdot \omega^i \}$$
Suppose all $u^i$ are quasi-concave.

**Theorem**

$\forall \alpha \in (0, 1], \exists \text{ an } \alpha\text{-slack Walrasian equilibrium } (x, p)$. Moreover, if all $u^i$ are semi-strictly quasi-concave, then $x$ is PO.
HZ Example
$\omega^i = (\frac{6\alpha}{1+2\alpha}, 0, 0)$
Having $\alpha > 0$ gives an “extra role” for prices, allowing agents 1 and 2 to spend above the income they get from $p \cdot \omega^i$. 
That doesn’t mean that $\omega^i$ don’t matter ... 

Suppose all $u^i$ are semi-strictly quasi-concave.

**Theorem**

\[ \forall \varepsilon > 0 \ \exists \ \alpha \in (0, 1] \text{ and an } \alpha \text{-slack Walrasian equilibrium } (x, p), \text{ s.t} \]

\[ x \text{ is PO and } \max\{u^i(y) : y \in \Delta_- \text{ and } p \cdot y \leq p \cdot \omega^i\} - u^i(x) < \varepsilon. \]

In particular, $x$ is $\varepsilon$-individually rational.
Let \((x, p)\) be an \(\alpha\)-slack Walrasian equilibrium.

If \(i\) envies \(j\) at \(x\), then \(p \cdot \omega^j > p \cdot \omega^i\).

“Society” values \(j\)’s endowment more than \(i\)’s.
Suppose all $u^i$ are concave and $C^1$. Let $(x, p)$ be an $\alpha$-slack Walrasian equilibrium. Denote by $S = \{i : u^i(x^i) = \max\{u^i(z^i) : z^i \in \Delta_-\}\}$ the set of satiated consumers, and by $U = [N] \setminus S$. Suppose that $\sum_{i\in U} x^i \gg 0$.

**Theorem**

*If $i$ envies $j$ at $x$ then $p \cdot \omega^j > p \cdot \omega^i$,*
Suppose all $u^i$ are concave and $C^1$. Let $(x, p)$ be an $\alpha$-slack Walrasian equilibrium. Denote by $$S = \{ i : u^i(x^i) = \max\{ u^i(z^i) : z^i \in \Delta_- \} \}$$ the set of satiated consumers, and by $U = [N] \setminus S$. Suppose that $\sum_{i \in U} x^i \gg 0$.

**Theorem**

If $i$ envies $j$ at $x$ then $p \cdot \omega^j > p \cdot \omega^i$, and $\exists$ welfare weights $\theta \in \mathbb{R}^U_{++}$ s.t

$$v(t) = \sup \{ \sum_{i \in U} \theta^i u^i(\tilde{x}^i) : (\tilde{x}^i) \in \Delta^U \text{ and }$$

$$\sum_{i \in U} \tilde{x}^i \leq \bar{\omega} + t (\omega^i - \omega^j) - \sum_{i \in S} x^i \},$$

then $(x^i)_{i \in U}$ solves the problem for $v(0)$, and $v(t) < v(0)$ for all $t$ small enough.
Idea

Classical result relies on Walras Law: $p \cdot z(p) = 0$ for all $p$. Walras Law does not hold in our model because...
Demand is not responsive to price once boundary is reached.
Budget constraint:

\[ p \cdot x^i \leq \alpha + (1 - \alpha) p \cdot \omega^i \]
Budget constraint:

\[ p \cdot (x^i - \omega^i) \leq \alpha(1 - p \cdot \omega^i). \]

This allows prices to matter: large prices imply that the value of excess demand is \(< 0\).
Consider $\phi : [0, \bar{p}]^L \rightarrow [0, \bar{p}]^L$ defined by

$$\phi_l(p) = \{\min\{\max\{0, \zeta_l + p_l\}, \bar{p}\} : \zeta \in z(p)\}.$$ 

where $\bar{p}$ is a large price.

**Lemma**

$\phi$ is upper hemi-continuous, convex- and compact- valued.

(In paper deal with a different $\phi$, which ensures PO.)
By Kakutani, \( \exists p^* \) and \( \zeta \in z(p^*) \) s.t

\[
p_i^* = \min \{ \max \{ 0, \zeta_i + p_i^* \}, \bar{p} \}.
\]

**Lemma**

\( p^* \cdot \zeta \geq 0. \)

This is sort of a “weak Walras law.”

**Pf:** \( \zeta_i < 0 \implies p_i^* = 0 \)
Lemma

$p^*_i < \bar{p}$ for all $l \in [L]$

Pf: Suppose $p^*_i = \bar{p}$. $\bar{p}$ is large $\implies 1 - p \cdot \omega^i < 0$; so $p \cdot (x^i - \omega^i) < 0$.

By adding up we get that

$$p \cdot \zeta \leq \alpha(N - p \cdot \bar{\omega}) < 0,$$

in contradiction to prev. lemma.
Now think about:

\[ p^*_i = \min\{\max\{0, \zeta_i + p^*_i\}, \bar{p}\}. \]

when \( p^*_i < \bar{p} \).

Either \( \zeta_i = 0 \) or \( \zeta_i \) and \( p^*_i = 0 \). Latter case is not possible.
Two agents and two schools.

\[
\begin{array}{c|cc}
    i & u^{i}_{s_1} & u^{i}_{s_2} \\
    \hline
    1 & 1 & 1 \\
    2 & 1 & 100 \\
\end{array}
\]

Endowments \( \omega^i = (1/2, 1/2) \).

Consider the allocations
\( x = ((1, 0), (0, 1)) \) and
\( y = ((1/2, 1/2), (1/2, 1/2)) \).
Note that \( x \) Pareto dominates \( y \).
### Pareto ranked equilibria

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>allocation</th>
<th>$p$</th>
<th>$\alpha + (1 - \alpha)p \cdot \omega^i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$x$</td>
<td>$(1, 1)$</td>
<td>1</td>
</tr>
<tr>
<td>1/2</td>
<td>$x$</td>
<td>$(1, 1)$</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>$y$</td>
<td>$(0, 1)$</td>
<td>1/2</td>
</tr>
<tr>
<td>1/2</td>
<td>$y$</td>
<td>$(0, 2)$</td>
<td>1</td>
</tr>
</tbody>
</table>
Theorem

Any Walrasian equilibrium with slack is weakly Pareto optimal, and any Walrasian equilibrium with slack and the cheapest-bundle property is Pareto optimal.
Related Literature

- Justified envy w/endowments: Yilmaz (2010)
- Exog. and endog. budgets: Mas-Colell (1992) and Le (2017)
- Endowments in school choice: Hamada, Hsu, Kurata, Suzuki, Ueda, and Yokoo (2017)

More references in the paper...
Conclusions

- New criteria for discrete (probabilistic) allocation.
- Model with *explicit* property rights.
- Fairness among unequals.
- Market equilibrium
- Fairness, efficiency and property rights.
The Leverage Cycle, Credit Surfaces, & Central Banking

John Geanakoplos

Based on
Geanakoplos-Zame (2014)
Ben Ami-Geanakoplos (2017)
Basic Leverage Cycle

On new loans.

Leverage Up — Asset Prices Up

Leverage Down — Asset Prices Down

Opposite with old loans.
Leverage Cycle Precursors

**Boom-Bust Story**

Minsky (1977), Kindleberger (1978)

No model or mathematics in either story.

No collateral. (Definition of leverage is debt to earnings.)

Driven by extrapolative (irrational) expectations.
Collateral

2) Holmstrom-Tirole 1997: Also corporate finance approach
3) Kiyotaki-Moore 1997: Canonical model of macro. $\infty$-horizon, two agents. These three papers changed macro. But

They ignored changes in leverage.


Focused on how leverage is determined.
Outline

1) Collateral Equilibrium: Endogenous Leverage, Credit Surface
2) Volatility and Leverage Binomial Economies
3) Leverage and Asset Prices
4) Leverage Cycle Mechanisms
5) Leverage Cycle 2003
6) 2007-09 Crisis: Test Case for the Leverage Cycle
7) Central Banking: Credit Surface in Practice
Collateral Equilibrium
(Geanakoplos 1997)
See also Geanakoplos-Zame 2014

Just as general as GE or GEI
no ad hoc costs
no ad hoc information asymmetries

Agents default unless they put up collateral.

Goods are durable (stocks and flows)

Default endogenous

Collateral is endogenous
Endogenous Leverage

How can one supply equals demand equation determine both the:

loan price (i.e. interest rate)

and the

collateral rate (i.e. leverage) for new loans?
Endogenous Leverage

Standard Economic Theory:

Equilibrium (supply = demand) determines interest rate.

In my theory:

Equilibrium determines Required Collateral or Leverage as well.
Redefine financial promises

As pair:

(Promise, Collateral)

with a different, competitive price for each pair.

Can imagine many alternative promises with same collateral.

Each has its own competitive price in collateral equilibrium.
Credit Surface
(Geanakoplos 1997)

All contracts $j$ offered on market simultaneously. Each contract trades at its own equilibrium price.

Lenders decide which ones to buy and borrowers decide which ones to sell. Nothing bilateral.

In equilibrium, every contract market must clear, and have same amount of buys and sells (often zero on both sides).
Endogenous Leverage: lenders and borrowers separately choose where on the credit surface they want to trade, taking \( \text{LTV} \) and \( r \) as given. Rational choice of contract, given (rational) expectations about future...
Credit Surface and Tight Credit

In the old interest rate centric macroeconomics, what does tight credit mean?

Tight credit doesn’t mean too high riskless interest rate.

Tight credit means a steep credit surface. Agents can’t borrow more unless they pay higher interest.
Outline

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Volatility and Leverage

Low Volatility ———> High Leverage
Leverage and Volatility

Wall Street practitioners know that higher volatility implies higher margins.

But I believe I was the first to point out, in 2003, that for the macroeconomy, higher uncertainty means tighter credit conditions, in a theorem.

This came before the crisis of 2007-09.

Brunnermeier and Pedersen 2009 also emphasized link between volatility and credit conditions.
Commodities Market History

Margin Requirement vs Realized Volatility

\[ y = 0.1082x + 0.0329 \]

\[ R^2 = 0.3726 \]
Binomial Economies

Arbitrary endowments
Arbitrary utilities
One risky asset $Y$
Serving as collateral

$Y$

$s=U$

$d_U$

$s=0$

$d_D < d_U$

$s=D$
Asset Y Payoff

Debt contract promise $j < j^*$

Family of debt contracts
Net payoff of buying asset on margin through debt contract $j < j^*$

Debt contract $j < j^*$ delivery

Family of debt contracts
Asset Y Payoff

Family of debt contracts

Debt contract promise $j > j^*$

$U$

$d_U$

$d_D$

$D$

45°
Net payoff of buying asset on margin through contract $j > j^*$: Arrow U security $(d_u - j, 0)$

Debt contract promise $j > j^*$

Debt contract j delivery

$45^\circ$
Asset Y Payoff

Family of debt contracts

Max min contract $j^*$
Borrower betting on $U$

Net payoff of buying asset on margin through contract $j^*$: Arrow $U$ security $(d_U - j, 0)$

Max min contract $j^*$

Family of debt contracts
Endogenous Leverage and Default in Binomial Economies

Fostel-Geanakoplos (2015)

True with all binomial trees, multiple assets, arbitrary preferences, arbitrary endowments.

Special risk-neutral case proved in Geanakoplos (2003)
Financial Assets vs non-Financial Assets

Financial Asset = Agents get no direct utility from holding the asset and productivity of the asset is independent of owner.

e.g. Shares of General Electric stock

e.g. Securitized Mortgage

e.g. Collateral on Repo Market

Non-Financial Asset = Agents get direct utility from holding asset, or productivity of asset depends on owner.

e.g. House

e.g. Factory that depends on manager

Literature has mixed together these two kinds of collateral.
Binomial MaxMin-No-Default Theorem with Financial Assets

If two states and financial assets, only non-contingent promises

Then in equilibrium everyone lends and borrows via one

maxmin contract = Biggest promise with no default.

Not true for non-financial collateral, like houses
Repo market vs mortgage market
Not true if asset is more productive for some than others
Not true for more than two states
Will never happen with financial assets!

U

Asset Y
Payoff

Arrow U
security

Space of debt contracts

Debt contract promise \( j > j^* \)

D

Contract Promises and Deliveries
Will always trade maxmin with financial assets!

Net payoff of buying asset on margin through contract $j^*$:
Arrow U security $(d_U - j, 0)$
Max min contract $j^*$

So no default in binomial economies with financial assets. But possibility of default determines level of credit.
Everybody chooses to trade at A if collateral is a financial asset.
Credit Surface in Binomial Economies

Greater Leverage due to Reduced down risk of collateral payoffs: $A \rightarrow A$.

$LTV_{j^*} = \frac{d_D/p}{1+r}$
Volatility and Leverage

Binomial Economy Theorem

Low Down Risk → High Leverage

For financial assets
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Consequences of Leverage and Changing Leverage
Leverage and Collateral Prices

Higher Leverage  \rightarrow  Higher Asset Prices

If interest rates the same

WHY?

Leverage boosts demand with heterogeneity.
Leverage and Asset Prices

In 2003 I made this idea the central theme of the leverage cycle.

As leverage goes up and down, asset prices go up and down.
Leverage and Asset Prices

Nobody had said this before (except possibly Minsky, who had no model).

Other work on collateral also did not have this idea, because they didn’t mention changing leverage.

Bernanke-Gertler (1996) and Kiyotaki-Moore (1997) had positive feedback between asset prices and borrowing (credit cycles) but they never considered the effect of leverage on asset prices.
Leverage and Asset Prices

Can Show this:

Across Equilibria Or Inside the Same Equilibrium

As in Geanakoplos 2003 in special risk neutral case

In 2014 Fostel-Geanakoplos proved theorem
Fostel-Geanakoplos 2014:

3 Binomial Economies with same (arbitrary) utilities and endowments, where one risky asset is a financial asset.

Economies defined by Contracts Available

N-economy: No contracts

L-economy = Leverage Economy: All non-contingent promises collateralized by Y

Arrow-Debreu economy
Fostel-Geanakoplos 2014:

Theorem: If the interest rate is the same in all three economies, then the equilibrium price of $Y$ and production of $Y$ is higher in the L-economy than in the N-economy.

Further, if agents have no future endowments of goods, other than output of period 0 assets, then the equilibrium price of $Y$ and production of $Y$ is higher in the L-economy than in the Arrow-Debreu economy.

Need agent heterogeneity for strictly higher.
Second result more surprising

Paradox: not only is Leverage price higher than No leverage price, but Leverage price can be higher than Arrow-Debreu price.

Agents constrained from borrowing to demand more Y because of lack of trust.

But in a model with complete trust, where everyone could borrow as much as he wants by selling any promise, there could be less demand for Y, despite access to more credit.
Collateral Value

If want to borrow, need to hold a house.

If want to bet on U, need to leverage a house purchase.

In Arrow Debreu can borrow without holding any particular asset, and can purchase Arrow securities without holding any particular asset. So house price rises above efficient level because of constraint.

Housing. Education

Collateral value creates deviations from Law of One
Leverage and Pricing

Binomial Economy Theorem

Higher Leverage \rightarrow Higher Asset Prices

For financial assets, if Interest Rate Unchanged
Outline

1) Collateral Equilibrium:  
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5) Leverage Cycle 2003
6) 2007-09 Crisis: Test Case for the Leverage Cycle
7) Central Banking: Credit Surface in Practice
Leverage Cycle Idea

Low Volatility → High Leverage → High Prices
Dynamic Leverage Cycle

Low Volatility → High Leverage → High Prices

Shock that greatly worsens bad tail (Volatility-Leverage Mechanism)

High Volatility → Low Leverage → Low Prices
Static Leverage Cycle

Low Volatility → High Leverage → High Prices

With Heterogeneous agents, High Debt means Fragile Economy.

So small shock to expected payoffs can cause big price crash

Wealth redistribution mechanism
Debt and Fragility: Wealth Redistribution Mechanism

1) Heterogeneity is key

2) Agents who like “houses” the most will buy them leveraged

3) If debt is big enough, or shock is big enough, they will have to sell a lot of houses to repay loans

4) But then a small decline in housing prices when they are selling makes them much poorer, reducing, rather than increasing, their housing demand
Debt and Fragility: Wealth Redistribution

5) If borrowers have much higher MPC housing (which is why they borrowed) then aggregate net income effect counteracts substitution effect.

6) Equilibrium becomes fragile. A small shock to quantities might require a big change in price to restore equilibrium.

7) With still bigger debt, get multiple equilibria.
Given any smooth Walrasian economy and any interior Pareto optimal allocation. Suppose there is some good and two agents who have different marginal propensities to consume (MPC) the good at the Pareto allocation.

**Heterogeneity and Enough Debt implies Fragility.** Then there is a reallocation of endowments giving rise to the Pareto optimal allocation as a fragile Walrasian equilibrium. The reallocation might involve debt.

**Heterogeneity and More Debt Implies Multiplicity** Extending the net trades even further gives multiple equilibria.
Thinking Outside the Edgeworth Box

Demand stable at no trade
Thinking Outside the Edgeworth Box

Demand gets flatter if A has higher MPC Y than B.
Debt: Thinking Outside the Edgeworth Box

Demand very flat
→ Fragile Equilibrium
Demand upward sloping → Multiple Equilibria
Leverage Cycle

Combines both mechanisms

Low Volatility → High Leverage → High Prices

High Debt means Fragile Economy

Small shock that worsens bad tail (scary bad news)

High Volatility → Low Leverage → Low Prices
VIX Daily Returns vs SPY Daily Returns

Scary Bad News
(Bad news that Increases volatility)
Is normal
Leverage Cycle

- Long Period of Low Volatility and Financial Innovation leads to Increased Leverage & Laxer Credit Standards
- That raises asset prices and Increases Activity.
- And makes economy more vulnerable. Borrowing\(^2\)
- (little) Bad news decreases all valuations.
- Most leveraged = most enthusiastic buyers lose most
- That lowers asset prices more
- Then increased uncertainty lowers leverage
- CDS suddenly appearing lowers prices more
- Central banks should smooth leverage cycle.
  - Restrain leverage in booms
Leverage Cycle

- Seems like common sense.
- Yet major implications still not accepted, even if more attention paid to it:
  - Empower Central Banks to regulate leverage
  - Put endogenous leverage – credit surface into large macro models
  - De-Stigmatize Default and Forgiveness
Shakespeare got this Right 400 years ago.

The Merchant of Venice

Who can remember the interest rate Shylock charged Antonio and Bassanio? Bassanio is no fool.

Quality of Mercy
Related Ideas

1) Limits to arbitrage and marginal buyers is related to Schleifer-Vishny (1997).

2) Also related to the notion of margin calls and margin spirals developed later in Brunnermeier-Pedersen (2009), including possibility of multiple equilibria.
3) Also related to later work of Bloom (2009) on uncertainty shocks.

4) Heterogeneity is modeled by Geanakoplos (2003) as different priors. This is related to literature on asset prices and heterogeneous beliefs in Harrison-Kreps (1978), Abreu-Brunnermeier (2003), and Scheinkman-Xiong (2003).
Heterogeneity Consistent with Leverage Cycle

Natural Buyers vs Public

- Differ in risk tolerance.
- Difference in future endowments, so different hedges
- Might use assets for production.
- Might get higher utility for holding assets (houses)
- Differ in ability to hedge.
- Differ in sophistication and knowledge.
- Or just more optimistic (different priors)
Heterogeneous Beliefs

Leverage Cycle does not rely on irrationality.

It relies on heterogeneity.

Geanakoplos (2003) had optimists and pessimists, but these are based on different priors.

All agents correctly update by Bayes rule.
Heterogeneous Beliefs

Contrast with Shiller irrational exuberance of investors or Shleifer et al.

I agree that there are psychological elements. This just makes the leverage cycle bigger.

Shiller suggested to me an interpretation of the leverage cycle as irrational exuberance of lenders, instead of animal spirits of investors.
Behavioral Models of Leverage

Thurner-Farmer-Geanakoplos (2012) is a behavioral model of the leverage cycle in which lenders use past volatility to set margins. So volatility ↓ → leverage ↑ → prices ↑ link is there.

Different “funds” use different levels of leverage. Funds get more money after they do well. So rising prices causes most leveraged firms to get more money, and raises overall leverage.

Assume funds have more stable beliefs than public. Then rising prices enriches leveraged funds and so lowers volatility. So

Behavioral model also explains reverse leverage cycle, prices ↑ → leverage ↑, volatility ↓, making endogenous feedbacks.

Leverage then causes fat tails and clustered volatility, because of margin calls.
Basic Leverage Cycle

Leverage Up  Asset Prices Up

Leverage Down  Asset Prices Down

Key driver of leverage could be rationally anticipated volatility.

Key driver could be extrapolative beliefs
Leverage Cycle vs Credit Cycle

The Leverage Cycle is not the same as a Credit Cycle.

A Leverage Cycle is a feedback between asset prices and leverage. A credit cycle is a feedback between asset prices and borrowing.

A Leverage Cycle always produces a Credit Cycle, but not vice versa.
Credit Cycles vs Leverage Cycles

The classical models of macroeconomic financial frictions produce credit cycles, not leverage cycles. In Kiyotaki-Moore (1997), to the extent leverage moves at all, it is counter cyclical.

Gertler-Kiyotaki, Brunnermeier-Sannikov all have counter cyclical leverage. In demand driven leverage, or moral hazard driven leverage, when things are bad the return available to borrowers rises, so they want to leverage more and the lenders think they care about the business more and so feels safer giving them a higher leverage (not bigger loan, but bigger ratio of loan to equity).
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John Geanakoplos, 2003
Proceedings of Econometric Society 2000 World Congress
(long before crisis of 2007-09)
All the Ingredients are Here

1) Collateral

2) Endogenous Leverage

3) Counter-cyclical volatility (scary bad news)

4) Optimists vs Pessimists

5) Leverage Cycle Dynamic with both wealth redistribution and volatility-leverage mechanism
3 Period C–Model

Y pays dividends at end

X pays 1 at end for sure

At U volatility of asset value is zero and expectation is high
At D volatility of asset value is high and expectation is low
Counter-cyclical volatility.
Consumers: Need heterogeneity.
Simple case:
\[ H = \{ h \in [0,1] \} \text{ continuum} \]

All risk neutral

Only care about terminal consumption

Probability of up is \( h \) for each consumer \( h \) so higher \( h \) means more optimistic.

Endowments for every \( h \in H \) of 1 unit of \( X \) and 1 unit of \( Y \) at 0 and nothing else.
Loans and Collateral

At each non-terminal node, agents can use 1 unit of $Y$ to collateralize any non-contingent promise $(j,j)$ in the immediately succeeding states.

Question:

What will be the price of $Y$ relative to $X$ at each node?
So what is price at 0 and D?
Crash at D
Crash really bad; news not.
Marginal buyer at 0
= .87.
Marginal Buyer at
D = .61

Crash really bad; news not.
Price Drop of 26 points from 0 to D is bigger than any agent thinks is justified by the news.

Agent $h = 1$ doesn’t change his mind at all.

Marginal buyer $h = .87$ thought it was worth $(1-(1-.87)^2)1 + (1-.87)^2(.2) = .98$ at 0 and then $.87(1) + .13(.2) = .90$ at D, which is a drop of 9 points.

Two thirds of drop in price comes from drop in marginal buyer rather than from bad news.
Leverage Cycle

At 0 lots of optimists, and leverage is high because uncertainty about prices at time 1 is not so big.

Crash at D because:

1) get a little bad news about payoffs of Y.

2) optimists (people with high marginal propensity to buy Y) lose money, and lose lots of it because they were so leveraged into Y at 0.

3) Leverage at D goes way down because uncertainty about value of Y is much higher.
Fall in price due more to drop in marginal buyer than shock.
Leverage Cycle Crashes Always Have same three aspects plus feedback:

Bad news makes everyone value assets less. But bad news is also scary, creating more uncertainty and more disagreement = high volatility

Leveraged buyers (optimists) crushed, some go bankrupt, others insolvent and functioning poorly.

De-leveraging because nervous lenders ask for more collateral
Leverage makes the asset price higher than it would have ever been without leverage.

But the low is lower than it would have been without leverage.

The gap between high and low is thus much bigger than it would have been.
Irrational Exuberance?

Agents foresee the crash and what will happen. Not irrational exuberance.

Why don’t investors who foresee the crash, when returns will really be high since the price will be so low, simply wait for the crash?

Some do. The Warren Buffets of the world. But there aren’t enough of them to prevent the crash.

h=.86 thinks asset is worth .98 but doesn’t buy at .95
Price falls more than any agent thinks it ought to because marginal buyer changes
Maturity Mismatch?

If long loans available at 0, wouldn’t they be traded, eliminating the crash?
Maturity Mismatch

Short term borrowing on long term asset

Optimists at D still think asset will do well, but because their loans come due, they must sell, which causes the crash

If had long term loans, they wouldn’t have had to sell

But they wouldn’t take them at time $0$ if they were available! Because by binomial no default theorem, then can borrow so little long -- only $0.2$. 
Multiple Leverage Cycles

Fostel-Geanakoplos (2008) again written before the 2007-09 crisis

Leverage cycle in one asset can create boom and bust in another asset, i.e. contagion. If same buyers in two markets, crash in one market means more opportunity in first and less wealth to spend in second.

One aspect of the contagion is the flight to collateral when credit gets tight (an explanation of flight to quality)
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Need to:

1) show boom and crash in prices
2) show parallel boom and bust in leverage
3) show parallel countercyclical changes in volatility
4) identify the little bit of scary bad news that triggered the crisis
Leverage and mortgages securities pricing
Leverage and housing prices

Note: Observe that the Down Payment axis has been reversed, because lower down payment requirements are correlated with higher home prices. For every AltA or Subprime first loan originated from Q1 2000 to Q1 2008, down payment percentage was calculated as appraised value (or sale price if available) minus total mortgage debt, divided by appraised value. For each quarter, the down payment percentages were ranked from highest to lowest, and the average of the bottom half of the list is shown in the diagram. This number is an indicator of down payment required: clearly many homeowners put down more than they had to, and that is why the top half is dropped from the average. A 13% down payment in Q1 2000 corresponds to leverage of about 7.7, and 2.7% down payment in Q2 2006 corresponds to leverage of about 37. Note Subprime/AltA Issuance Stopped in Q1 2008. Source: Geanakoplos (2010).
Figure 1
Household leverage ratios: Debt to disposable income


Glick-Lansing FRBSF 2009
Figure 3
Household leverage and the run-up in house prices

Note: The plotted line depicts the best fit relationship in the data as generated by a simple least square statistical regression.

Leverage is debt to equity in this San Francisco Fed study
Glick-Lansing FRBSF 2009
Figure 4
Household leverage and the decline in consumption

% change in consumption, 08Q2-09Q1

% point change in household leverage, 1997-2007

Note: The plotted line depicts the best fit relationship in the data as generated by a simple least square statistical regression.

Glick-Lansing FRBSF 2009
VIX levels vs SPY (demeaned)
How did crash start?

Conventional view is that housing prices suddenly fell, and fell more than anyone imagined, so banks lost huge money, and that rippled through economy.

My view: Housing prices had been going up because of increasing leverage, but LTV can’t go above 100, so increase bound to stop as LTV approached 100.

Scary bad news of delinquencies + credit default swaps creation in mortgages at top of cycle led to dramatic fall in BBB prices before big fall in housing prices.

Led to tightening of collateral on houses. That led to dramatic fall in housing prices. Then government did not intervene properly in housing market, and prices fell further.
Look More Closely at Timing

Housing Peak at Q2 2006
Slightly down Q4 2006
CDS created on subprime late 2005
ABX securities index collapses Jan 2007
Then housing prices start to free fall
ScaryBad News

OTD Delinquent 90+ / Orig

Jan-03  May-03  Sep-03  Jan-04  May-04  Sep-04  Jan-05  May-05  Sep-05  Jan-06  May-06  Sep-06  Jan-07  May-07  Sep-07

Bank Runs vs Leverage Cycles

Bank run or Panic is popular narrative for crisis of 2007-09. See Geithner and Bernanke books.

Popularized by Gorton, who said crisis reminiscent of bank panics of 1907 and before.

But bank runs like Diamond-Dybvig are based on “tragedy of the commons” logic. If everyone else is pulling money out of underwater bank, then I should too in order to get pro rata share instead of nothing.

Collateral is not a public resource but a private resource.
Anticipation of disaster causes crash.
Bank Runs vs Leverage Cycles

But flavor of bank runs can be found in collateral equilibria if there are multiple equilibria, without common resource.

Maybe at D there might be multiple equilibria? If everyone thinks Y is worth more at D, then optimists won’t have to sell as much to pay off debt, and then the price can turn out to be higher?
Bank Runs vs Leverage Cycles

In leverage cycle example just presented, with linear demand, equilibrium is unique.

But with different preferences can get multiplicity. Assume all agents $h \in [h_1, h_0]$ think $\text{prob}(D) = 1$ but $\text{prob}(DU) = 1$. They all behave like old agents in equilibrium, but now get a new equilibrium if people expect old equilibrium when they get down to $D$, then gov announces price will be higher.
Bank Runs vs Leverage Cycles

Brunnermeier and Pedersen (2009) had multiplicity, in special model.

Student Yaniv Ben Ami had key idea. The bigger is leverage, the more likely is multiplicity.
Bank Runs vs Leverage Cycles

Slutsky Equation:

\[ \frac{Dy}{dp} = \sum_i \frac{dy_i}{dp} \mid \text{subs} + \sum_i MPC_i(e_i - y_i) \]

So if agents with high marginal propensity to consume \( y \) are being forced to sell it, the income effect can reverse the substitution effect. Get upward sloping demand, hence multiplicity.

High enough debt creates multiplicity. But was debt that high in 2007-08?
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Credit Surface in Practice

Lenders worry about collateral. Also worry about reliability and income of borrowers. Though macroeconomists have not made much of the credit surface, real world investors do all the time. In practice, interest rates depend on many credit-quality indicators.

Credit quality can be measured directly in terms of ratings for corporations or FICO score for individual borrowers. In addition one might expect debt payment to income or debt to wealth to be important variables in determining the loan rate.
Credit Surface Changes over Time

These changes indicate whether credit is getting tighter or looser.

And whether the economy is cold or hot.
Different Credit Surfaces

David Rappoport (Washington Fed) – John Geanakoplos
2006-6, 30-year conventional purchase mortgages

Notes: 30-year, 1st lien, fixed non-IO, purchase, conventional, FNMA and FHLMC Mortgages.
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Notes: 30-year, 1st lien, fixed non-IO, purchase, conventional, FNMA and FHLMC Mortgages.
2016-12, 30-year conventional purchase mortgages

Notes: 30-year, 1st lien, fixed non-IO, purchase, conventional, FNMA and FHLMA Mortgages.
2000-6, 7-10-year corporate bonds

Notes: domestic non-financial firms, 7-10-year maturity, rating \( \geq B- \), total debt at least total bond debt, OAS in \([5, 3,500]\), estimates use at least 45 bond-day observations per rating-leverage bins.
2002-9, 7-10-year corporate bonds

Notes: domestic non-financial firms, 7-10-year maturity, rating $\geq$ B-, total debt at least total bond debt, OAS in [5, 3,500], estimates use at least 45 bond-day observations per rating-leverage bins.
Notes: domestic non-financial firms, 7-10-year maturity, rating ≥ B-, total debt at least total bond debt, OAS in [5, 3,500], estimates use at least 45 bond-day observations per rating-leverage bins.
2007-3, 7-10-year corporate bonds

Notes: domestic non-financial firms, 7-10-year maturity, rating $\geq$ B-, total debt at least total bond debt, OAS in [5, 3,500], estimates use at least 45 bond-day observations per rating-leverage bins.
2008-9, 7-10-year corporate bonds

Notes: domestic non-financial firms, 7-10-year maturity, rating ≥ B-, total debt at least total bond debt, OAS in [5, 3,500], estimates use at least 45 bond-day observations per rating-leverage bins.
2008-12, 7-10-year corporate bonds

Notes: domestic non-financial firms, 7-10-year maturity, rating ≥ B-, total debt at least total bond debt, OAS in [5, 3,500], estimates use at least 45 bond-day observations per rating-leverage bins.
2009-6, 7-10-year corporate bonds

Notes: domestic non-financial firms, 7-10-year maturity, rating ≥ B-, total debt at least total bond debt, OAS in [5, 3,500], estimates use at least 45 bond-day observations per rating-leverage bins.
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2017-3, 7-10-year corporate bonds

Notes: domestic non-financial firms, 7-10-year maturity, rating ≥ B-, total debt at least total bond debt, OAS in [5, 3,500], estimates use at least 45 bond-day observations per rating-leverage bins.
2012Q4, Lending Club Loan Originations

Notes: 3-year loans.
Notes: 3-year loans.
2014Q4, Lending Club Loan Originations

Notes: 3-year loans.
Notes: 3-year loans.
Notes: 3-year loans.
2017Q4, Lending Club Loan Originations

Notes: 3-year loans.
Leverage and Borrower “Credit Quality”

Punishment is older incentive than collateral to induce repayment.

Can model punishment as utility loss $\lambda_h$ per (real) dollar defaulted by agent $h$. (Dubey-Geanakoplos-Shubik 1988). Proxy for FICO.

As real incomes rise, default punishment gets bigger relative to marginal utility of consumption, so incentive to default goes up in bad times, and goes down in good times.
Pro-cyclical lending terms

Hence rational lenders will toughen lending terms in bad times and loosen them in good times. Just like collateral constraints (assuming countercyclical volatility).
The Leverage Cycle

- Rising Asset Prices
- Improving Fundamentals
- Declining Volatility
- Loosening Lending Standards
The Leverage Cycle, Cont’d.

- Falling Asset Prices
- Worsening Fundamentals
- Rising Volatility
- Tightening Lending Standards
Credit Feedback Externality

Thus credit terms are usually pro-cyclical. With incomplete markets, this could create negative externalities.

In fact, that is precisely my understanding of the rationale for central bank interventions in the credit markets.
Credit surface and monetary policy

I would recommend that central banks and the OFR map out the credit surfaces for important collateral, and publish the surfaces quarterly.

I recommend that the central banks couch their analysis in terms of which parts of which credit surfaces they regard as too tight or loose, and what affect their interventions in the riskless interest rate will have on the whole credit surfaces.
Credit surface and monetary policy

Did the Fed anticipate that Quantitative Easing would dramatically loosen the corporate credit surface but not the mortgage credit surface?

I further recommend that central banks intervene directly in non-riskless parts of the credit surface, as the Bank of Israel did in limiting leverage on mortgages to 60% and as the Fed did in the crisis to make loans at higher LTV than the market was willing on credit card, auto loan, and student loan securities.
Debt Forgiveness

Marking loans to market.

Then consider debt relief.

Another policy tool.